Solving Exponential and Logarithmic Equations

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EXAMPLE 1 ***Solving by Equating Exponents***

Solve $4^{3x} = 8^{x+1}$.

SOLUTION

$4^{3x} = 8^{x+1}$	**Write original equation.**
$(2^2)^{3x} = (2^3)^{x+1}$	**Rewrite each power with base 2.**
$2^{6x} = 2^{3x+3}$	**Power of a power property**
$6x = 3x + 3$	**Equate exponents.**
$x = 1$	**Solve for *x*.**

✓**CHECK** Check the so

$4^{3 \cdot 1} \stackrel{?}{=} 8^{1+1}$

$64 = 64$ ✓

1) This is a "re-write with same base" problem. We did this on day 1!!

EXAMPLE 2 ***Taking a Logarithm of Each Side***

Solve $2^x = 7$.

SOLUTION

$2^x = 7$	**Write original equation.**
$\log_2 2^x = \log_2 7$	**Take $\log_2$ of each side.**
$x = \log_2 7$	**$\log_b b^x = x$**
$x = \frac{\log 7}{\log 2} \approx 2.807$	**Use change-of-base formula and a calculator.**

▶ The solution is about 2.807. Check this in the original equation.

2) & 3) To get rid of an Exponent… take the log of both sides! I will show how to do calculator in class.

EXAMPLE 3 ***Taking a Logarithm of Each Side***

Solve $10^{2x-3} + 4 = 21$.

SOLUTION

$10^{2x-3} + 4 = 21$	**Write original equation.**
$10^{2x-3} = 17$	**Subtract 4 from each side.**
$\log 10^{2x-3} = \log 17$	**Take common log of each side.**
$2x - 3 = \log 17$	**$\log 10^x = x$**
$2x = 3 + \log 17$	**Add 3 to each side.**
$x = \frac{1}{2}(3 + \log 17)$	**Multiply each side by $\frac{1}{2}$.**
$x \approx 2.115$	**Use a calculator.**

EXAMPLE 5 ***Solving a Logarithmic Equation***

Solve $\log_3(5x - 1) = \log_3(x + 7)$.

SOLUTION

$\log_3(5x - 1) = \log_3(x + 7)$ **Write original equation.**

$5x - 1 = x + 7$ **Use property stated above.**

$5x = x + 8$ **Add 1 to each side.**

$x = 2$ **Solve for *x*.**

▶ The solution is 2.

✓**CHECK** Check the solution by substituting it into the original

$\log_3(5x - 1) = \log_3(x + 7)$ **Write original equation.**

$\log_3(5 \cdot 2 - 1) \stackrel{?}{=} \log_3(2 + 7)$ **Substitute 2 for *x*.**

$\log_3 9 = \log_3 9$ ✓ **Solution checks.**

5) This is a "log on both sides" problem. We did this on day 1 too!!

EXAMPLE 6 ***Exponentiating Each Side***

Solve $\log_5(3x + 1) = 2$.

SOLUTION

$\log_5(3x + 1) = 2$ **Write original equation.**

$5^{\log_5(3x+1)} = 5^2$ **Exponentiate each side using base 5.**

$3x + 1 = 25$ $b^{\log_b x} = x$

$x = 8$ **Solve for *x*.**

✓**CHECK** Check the solution by substituting it into th

$\log_5(3x + 1) = 2$ **Write original equation.**

$\log_5(3 \cdot 8 + 1) \stackrel{?}{=} 2$ **Substitute 8 for *x*.**

$\log_5 25 \stackrel{?}{=} 2$ **Simplify.**

$2 = 2$ ✓ **Solution checks.**

6) To get rid of a log… raise both sides with the base as an exponent!

EXAMPLE 7 ***Checking for Extraneous Solutions***

Solve $\log 5x + \log(x - 1) = 2$. Check for extraneous solutions.

SOLUTION

$\log 5x + \log(x - 1) = 2$ **Write original equation.**

$\log[5x(x - 1)] = 2$ **Product property of logarithms**

$10^{\log(5x^2 - 5x)} = 10^2$ **Exponentiate each side using base 10.**

$5x^2 - 5x = 100$ $10^{\log x} = x$

$x^2 - x - 20 = 0$ **Write in standard form.**

$(x - 5)(x + 4) = 0$ **Factor.**

$x = 5$ or $x = -4$ **Zero product property**

7) This is the toughest one. You have to CONDENSE first and then raise both sides to the exponent.

The solutions appear to be 5 and −4. However, when you check these in the original equation or use a graphic check as shown at the right, you can see that $x = 5$ is the only solution.

▶ The solution is 5.