7.2 Properties of Rational Exponents

What you should learn

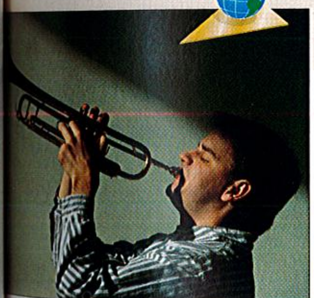
GOAL 1 Use properties of rational exponents to evaluate and simplify expressions.

GOAL 2 Use properties of rational exponents to solve **real-life** problems, such as finding the surface area of a mammal in **Example 8**.

Why you should learn it

▼ To model **real-life** quantities, such as the frequencies in the musical range of a trumpet for **Ex. 94**.

REAL LIFE



GOAL 1 PROPERTIES OF RATIONAL EXPONENTS AND RADICALS

The properties of integer exponents presented in Lesson 6.1 can also be applied to rational exponents.

CONCEPT SUMMARY — **PROPERTIES OF RATIONAL EXPONENTS**

Let a and b be real numbers and let m and n be rational numbers. The following properties have the same names as those listed on page 323, but now apply to rational exponents as illustrated.

	PROPERTY	EXAMPLE
1.	$a^m \cdot a^n = a^{m+n}$	$3^{1/2} \cdot 3^{3/2} = 3^{(1/2 + 3/2)} = 3^2 = 9$
2.	$(a^m)^n = a^{mn}$	$(4^{3/2})^2 = 4^{(3/2 \cdot 2)} = 4^3 = 64$
3.	$(ab)^m = a^m b^m$	$(9 \cdot 4)^{1/2} = 9^{1/2} \cdot 4^{1/2} = 3 \cdot 2 = 6$
4.	$a^{-m} = \frac{1}{a^m}, a \neq 0$	$25^{-1/2} = \frac{1}{25^{1/2}} = \frac{1}{5}$
5.	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{6^{5/2}}{6^{1/2}} = 6^{(5/2 - 1/2)} = 6^2 = 36$
6.	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$	$\left(\frac{8}{27}\right)^{1/3} = \frac{8^{1/3}}{27^{1/3}} = \frac{2}{3}$

If $m = \frac{1}{n}$ for some integer n greater than 1, the third and sixth properties can be written using radical notation as follows:

$$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b} \qquad \textbf{Product property}$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \qquad \textbf{Quotient property}$$

EXAMPLE 1 *Using Properties of Rational Exponents*

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Look Back
For help with properties of exponents, see p. 324.

Use the properties of rational exponents to simplify the expression.

a. $5^{1/2} \cdot 5^{1/4} = 5^{(1/2 + 1/4)} = 5^{3/4}$

b. $(8^{1/2} \cdot 5^{1/3})^2 = (8^{1/2})^2 \cdot (5^{1/3})^2 = 8^{(1/2 \cdot 2)} \cdot 5^{(1/3 \cdot 2)} = 8^1 \cdot 5^{2/3} = 8 \cdot 5^{2/3}$

c. $(2^4 \cdot 3^4)^{-1/4} = [(2 \cdot 3)^4]^{-1/4} = (6^4)^{-1/4} = 6^{[4 \cdot (-1/4)]} = 6^{-1} = \frac{1}{6}$

d. $\frac{7}{7^{1/3}} = \frac{7^1}{7^{1/3}} = 7^{(1 - 1/3)} = 7^{2/3}$

e. $\left(\frac{12^{1/3}}{4^{1/3}}\right)^2 = \left[\left(\frac{12}{4}\right)^{1/3}\right]^2 = (3^{1/3})^2 = 3^{(1/3 \cdot 2)} = 3^{2/3}$

EXAMPLE 2 Using Properties of Radicals

Use the properties of radicals to simplify the expression.

a. $\sqrt[3]{4} \cdot \sqrt[3]{16} = \sqrt[3]{4 \cdot 16} = \sqrt[3]{64} = 4$ **Use the product property.**

b. $\dfrac{\sqrt[4]{162}}{\sqrt[4]{2}} = \sqrt[4]{\dfrac{162}{2}} = \sqrt[4]{81} = 3$ **Use the quotient property.**

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For a radical to be in **simplest form,** you must not only apply the properties of radicals, but also remove any perfect *n*th powers (other than 1) and rationalize any denominators.

EXAMPLE 3 Writing Radicals in Simplest Form

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Write the expression in simplest form.

a. $\sqrt[3]{54} = \sqrt[3]{27 \cdot 2}$ **Factor out perfect cube.**

$= \sqrt[3]{27} \cdot \sqrt[3]{2}$ **Product property**

$= 3\sqrt[3]{2}$ **Simplify.**

b. $\sqrt[5]{\dfrac{3}{4}} = \sqrt[5]{\dfrac{3 \cdot 8}{4 \cdot 8}}$ **Make the denominator a perfect fifth power.**

$= \sqrt[5]{\dfrac{24}{32}}$ **Simplify.**

$= \dfrac{\sqrt[5]{24}}{\sqrt[5]{32}}$ **Quotient property**

$= \dfrac{\sqrt[5]{24}}{2}$ **Simplify.**

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Two radical expressions are **like radicals** if they have the same index and the same radicand. For instance, $\sqrt[3]{2}$ and $4\sqrt[3]{2}$ are like radicals. To add or subtract like radicals, use the distributive property.

EXAMPLE 4 Adding and Subtracting Roots and Radicals

Perform the indicated operation.

a. $7(6^{1/5}) + 2(6^{1/5}) = (7 + 2)(6^{1/5}) = 9(6^{1/5})$

b. $\sqrt[3]{16} - \sqrt[3]{2} = \sqrt[3]{8 \cdot 2} - \sqrt[3]{2}$

$= \sqrt[3]{8} \cdot \sqrt[3]{2} - \sqrt[3]{2}$

$= 2\sqrt[3]{2} - \sqrt[3]{2}$

$= (2 - 1)\sqrt[3]{2}$

$= \sqrt[3]{2}$

The properties of rational exponents and radicals can also be applied to expressions involving variables. Because a variable can be positive, negative or zero, sometimes absolute value is needed when simplifying a variable expression.

$\sqrt[n]{x^n} = x$ when n is odd	$\sqrt[7]{2^7} = 2$ and $\sqrt[7]{(-2)^7} = -2$
$\sqrt[n]{x^n} = \lvert x \rvert$ when n is even	$\sqrt[4]{5^4} = 5$ and $\sqrt[4]{(-5)^4} = 5$

Absolute value is not needed when all variables are assumed to be positive.

EXAMPLE 5 *Simplifying Expressions Involving Variables*

Simplify the expression. Assume all variables are positive.

a. $\sqrt[3]{125y^6} = \sqrt[3]{5^3(y^2)^3} = 5y^2$

b. $(9u^2v^{10})^{1/2} = 9^{1/2}(u^2)^{1/2}(v^{10})^{1/2} = 3u^{(2 \cdot 1/2)}v^{(10 \cdot 1/2)} = 3uv^5$

c. $\sqrt[4]{\dfrac{x^4}{y^8}} = \dfrac{\sqrt[4]{x^4}}{\sqrt[4]{y^8}} = \dfrac{\sqrt[4]{x^4}}{\sqrt[4]{(y^2)^4}} = \dfrac{x}{y^2}$

d. $\dfrac{6xy^{1/2}}{2x^{1/3}z^{-5}} = 3x^{(1 - 1/3)}y^{1/2}z^{-(-5)} = 3x^{2/3}y^{1/2}z^5$

EXAMPLE 6 *Writing Variable Expressions in Simplest Form*

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Write the expression in simplest form. Assume all variables are positive.

a. $\sqrt[5]{5a^5b^9c^{13}} = \sqrt[5]{5a^5b^5b^4c^{10}c^3}$ **Factor out perfect fifth powers.**

$= \sqrt[5]{a^5b^5c^{10}} \cdot \sqrt[5]{5b^4c^3}$ **Product property**

$= abc^2\sqrt[5]{5b^4c^3}$ **Simplify.**

b. $\sqrt[3]{\dfrac{x}{y^7}} = \sqrt[3]{\dfrac{xy^2}{y^7y^2}}$ **Make the denominator a perfect cube.**

$= \sqrt[3]{\dfrac{xy^2}{y^9}}$ **Simplify.**

$= \dfrac{\sqrt[3]{xy^2}}{\sqrt[3]{y^9}}$ **Quotient property**

$= \dfrac{\sqrt[3]{xy^2}}{y^3}$ **Simplify.**

EXAMPLE 7 *Adding and Subtracting Expressions Involving Variables*

Perform the indicated operation. Assume all variables are positive.

a. $5\sqrt{y} + 6\sqrt{y} = (5 + 6)\sqrt{y} = 11\sqrt{y}$

b. $2xy^{1/3} - 7xy^{1/3} = (2 - 7)xy^{1/3} = -5xy^{1/3}$

c. $3\sqrt[3]{5x^5} - x\sqrt[3]{40x^2} = 3x\sqrt[3]{5x^2} - 2x\sqrt[3]{5x^2} = (3x - 2x)\sqrt[3]{5x^2} = x\sqrt[3]{5x^2}$